

TECHNICAL MEMO



Date: October 5, 2026
To: PFC Documentation Set
From: David Potyondy
Re: Material Degradation Study [mmPkg9.4]
Ref: 5-8006:25R22

Petroleum extraction occurs in very weakly cemented sandstones, for which it is difficult to obtain intact cores for laboratory testing. A better understanding of the strength of this rock can be obtained by constructing a bonded-particle model and subjecting it to direct-tension and uniaxial compressive strength (UCS) tests while observing and measuring the material response. We begin with a model for competent dense sandstone and then degrade its microproperties, being guided by microstructural data of the physical rock. This memo summarizes the preliminary work that uses a 2D parallel-bonded material. It is planned to enhance the model to mimic a water-bearing sandstone by replacing the parallel-bonded material with a soft-bonded material, using the work of Zhu et al. (2021) as a starting point. After an acceptable 2D model is developed, then a corresponding 3D model will be developed.

1.0 PFC2D MODEL

The synthetic rock is a two-dimensional bonded-particle model employing the distinct-element method using the two-dimensional program PFC2D (ITASCA, 2025) to simulate the movement and interaction of rigid particles obeying Newton's laws of motion. The microproperties consist of stiffness and strength parameters for the particles and bonds. Damage is represented explicitly as broken bonds, which form and coalesce into macroscopic fractures when load is applied (see Figure 1). The modeling methodology (Potyondy and Cundall, 2004; Potyondy, 2015) has been applied extensively to model a variety of rock mechanics problems.¹

¹ Potyondy and Cundall (2004) has been cited 5,946 times (Google Scholar accessed 2 January 2025).

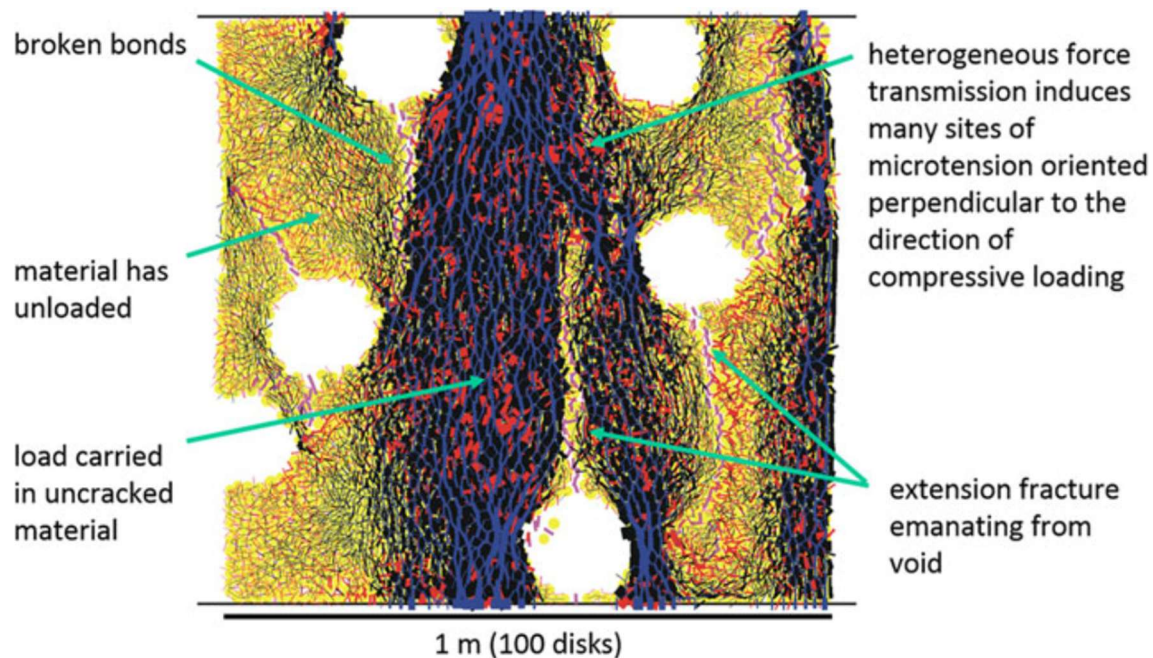


Figure 1 *A two-dimensional bonded-particle model consisting of a packed assembly of rigid disks joined by deformable and breakable cement at grain-grain contacts in the post-peak region of a UCS test. Blue and black denote compression and red denotes tension. (from Fig. 3 of Potyondy [2015])*

All models used in this study are two-dimensional parallel-bonded materials. A parallel-bonded material behaves like cemented rigid beads, which is a good approximation of sandstone for which most of the deformation occurs at the contacts and not in the grains (see Figure 2). The 2D model corresponds with a packing of unit-thickness disks (see Figure 3).

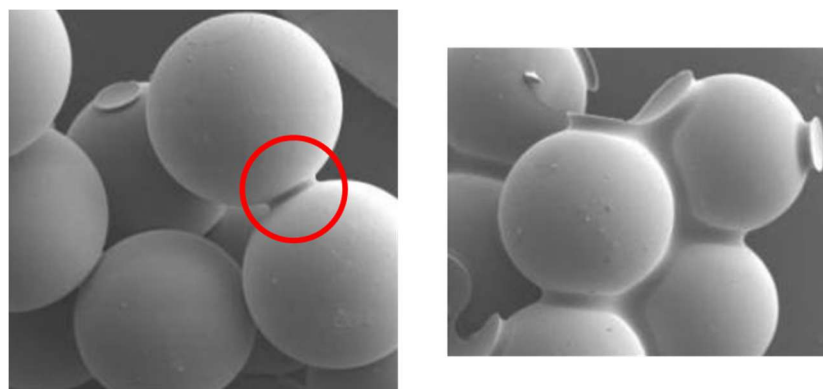


Figure 2 *Glass beads cemented with epoxy. The parallel-bonded material mimics this microstructure. The cement content is controlled by both the radius of the cement cylinder (circled in red) and the extent of pore filling.*

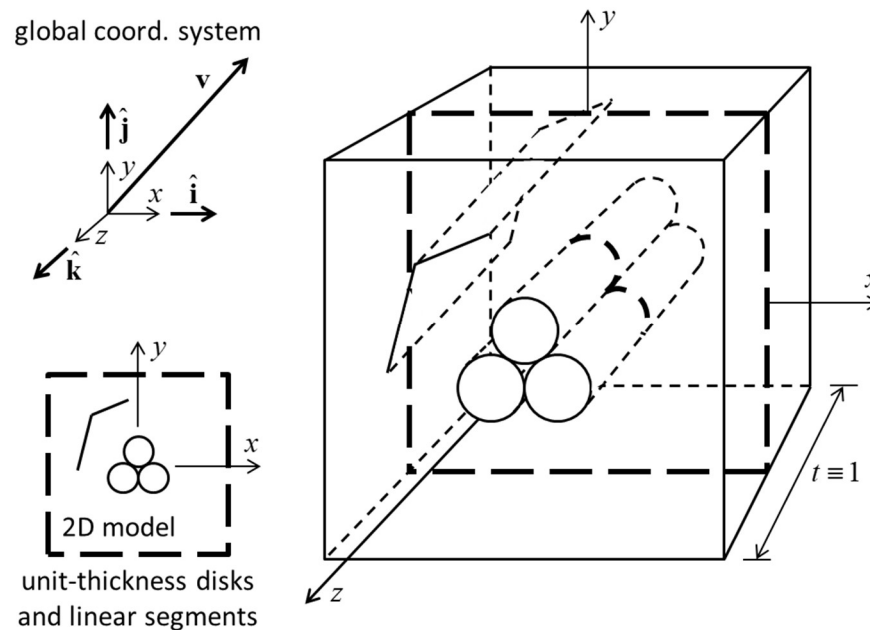


Figure 3 *Global coordinate system and orientation of the 2D model, which consists of unit-thickness disks and linear segments centered in the xy plane.*

A parallel-bonded material is a granular assembly in which the linear parallel bond contact model exists at all grain-grain contacts at the end of the packing phase; the linear contact model is assigned to new grain-grain contacts that may form during subsequent motion. A behavior summary of the linear parallel bond contact model follows — refer to ITASCA (2024) for a comprehensive description of the model.

The linear parallel bond model provides the behavior of two interfaces: an infinitesimal, linear elastic (no-tension) and frictional interface that carries a force, and a finite-size, linear elastic and bonded interface that carries a force and moment (see Figure 4). The first interface is equivalent to the linear model: it does not resist relative rotation, and slip is accommodated by imposing a Coulomb limit on the shear force. The second interface is called a parallel bond, because when bonded, it acts in parallel with the first interface. When the second interface is bonded, it resists relative rotation, and its behavior is linear elastic until the strength limit is exceeded and the bond breaks, making it unbonded. When the second interface is unbonded, it carries no load. The unbonded linear parallel bond model is equivalent to the linear model. The linear parallel bond model with inactive dashpots and a reference gap of zero corresponds with the parallel-bond model of Potyondy and Cundall (2004).

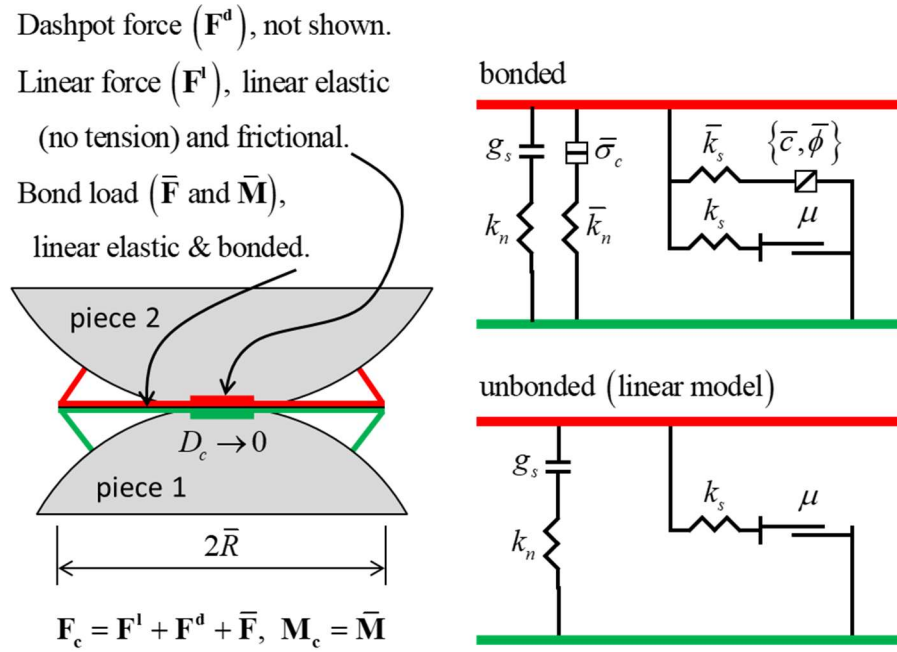


Figure 4 Behavior and rheological components of the linear parallel bond model with inactive dashpots.

Material genesis occurs within a material vessel (see Figure 5). Stress and strain are denoted by

$$\begin{aligned} \text{stress: } & \{\sigma_{xx}, \sigma_{yy}, \sigma_{zz}, \sigma_{xy}, \sigma_{xz}, \sigma_{yz}\} \\ \text{strain: } & \{\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{zz}, \varepsilon_{xy}, \varepsilon_{xz}, \varepsilon_{yz}\} \end{aligned} \quad (1)$$

where $\sigma_{ii} > 0$ is tension and $\varepsilon_{ii} > 0$ is extension. For the 2D model, the out-of-plane stress and strain components are equal to zero so that stress is $\{\sigma_{xx}, \sigma_{yy}, \sigma_{xy}\}$ and strain is $\{\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy}\}$. The following stress and strain quantities are defined.

- Axial and lateral stress and strain (2D model, $\varepsilon_{zz} \equiv \sigma_{zz} \equiv 0$):

$$\begin{aligned} \sigma_a &= \sigma_{yy}, & \sigma_l &= \sigma_{xx} \\ \varepsilon_a &= \varepsilon_{yy}, & \varepsilon_l &= \varepsilon_{xx} \end{aligned} \quad (2)$$

- Deviator stress and strain:²

² The deviator stress should not be confused with the deviatoric stress (Engelder, 1994). The amount by which the axial stress component departs from the confining pressure in a triaxial test is commonly called the differential stress in the rock mechanics and geological literature, and is used by Paterson and Wong (2005, p. 6) in the direct reporting of triaxial test results.

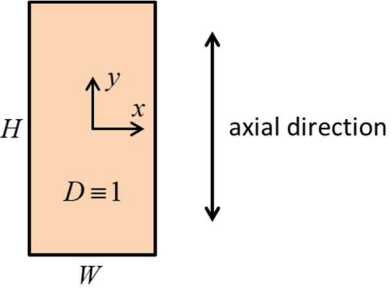
$$\sigma_d = \sigma_a - \sigma_l, \quad \varepsilon_d = \varepsilon_a - \varepsilon_l \quad (3)$$


Figure 5 *Material vessel in which the specimen is created by packing together the particles and then bonding them together, and coordinate system used to define stress and strain.*

The UCS test is performed by moving two walls that serve as loading platens toward one another at a constant axial strain rate that is slow enough to ensure quasistatic response. We denote this loading rate as the quasistatic loading rate $(\dot{\varepsilon}_a^{qs})$.³ The axial strain rate is related to the platen velocity as shown in Figure 6. The axial and lateral stresses are the average force of opposing walls divided by the corresponding cross-sectional area. There are no side walls in a UCS test so that the lateral stress is zero. The axial strain is based on the change of distance between opposing walls. The lateral strain is computed by integrating the strain-rate tensor over time. The strain-rate tensor is a best-fit that minimizes the error between predicted and measured velocities of all disks with centroids contained within a measurement circle.⁴ The final strain-rate tensor is taken as the average value from three overlapping PFC2D measurement circles aligned symmetrically along the specimen axis.

³ The quasistatic loading rate is the largest axial strain rate that gives quasistatic response meaning that reducing the axial strain rate below this value does not affect the stress-strain curve — see Potyondy (2025, Section 5.4: Loading Rate). If a specimen is loaded faster than this rate, it will exhibit dynamic effects whereby the peak force will be larger than the quasistatic value. The quasistatic loading rate of the synthetic material is much larger than that of a physical test, because the synthetic material is heavily damped and does not contain the same dissipative mechanisms as the physical material. For the models used in this study

$\dot{\varepsilon}_a^{qs} = 0.05 \text{ s}^{-1}$.

⁴ The measurement logic is described in ITASCA (2025).

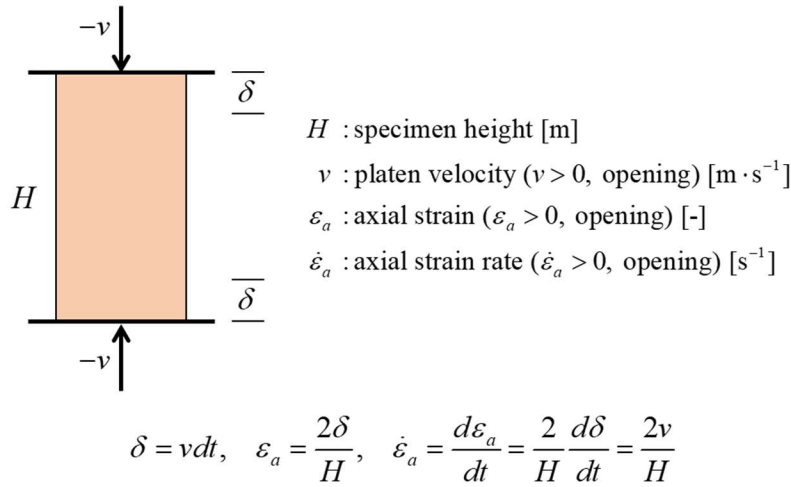


Figure 6 UCS test set up, along with definitions of axial strain, axial strain rate, and platen velocity.

The direct-tension test is performed by gripping the specimen at its ends and pulling it apart slowly while monitoring the axial stress and strain. The axial stress and strain are computed by averaging the forces (using the approach of Christoffersen et al. [1981]) and by integrating the strain-rate tensor over time, respectively, within three overlapping PFC2D measurement circles aligned symmetrically along the specimen axis.

Damage in the parallel-bonded material consists of bond breakages that are called cracks.⁵ A bond exists at the contact between two particles. A crack in the PFC2D model is a line segment of unit-thickness depth that is colored red or blue to denote tensile or shear failure, respectively. The crack size for a parallel-bonded material is two times the parallel-bond radius; thus, small cracks are associated with small cement bridges. The crack is located at the contact and is oriented perpendicular to the line joining the particle centers.

2.0 MATERIAL-DEGRADATION STUDY

This study is provided as part of the material-modeling support package (Potyondy, 2025) in the **ExampleProjects2D/matDegrade** directory. A well-cemented soft sandstone is modeled as a 2D parallel-bonded material and is used as the base material whose microstructure is degraded. The base material (with microproperties listed in Table 1) is created in a rectangular material vessel (with height and width of 100 and 50 mm, respectively, unit depth, and 3 GPa effective modulus). The grains are drawn from a uniform size distribution with diameters ranging from 0.8 to 1.2 mm, giving a specimen resolution of 50,⁶ which is sufficient to provide a representative volume of material. The grain-scaling packing procedure is used to pack the grains to a 30 MPa material pressure, and then parallel bonds are added between all grains that are in contact with one another (see Figure 7). The grain connectivity is quantified by the bond coordination number (c_b),

⁵ Crack monitoring is described in Potyondy (2025).

⁶ Specimen resolution (Φ_s) is defined as the number of grains across the specimen diameter.

which is the average number of intact bonds per grain (considering all grains including those on the boundary).

Table 1 Microproperties of Base Material

Property	Value
Common group	
N_m	base
$T_m, \alpha, C_p, \rho_v [\text{kg/m}^3]$	2, 0.7, 1, 1960
$S_g, T_{SD}, \{D_{\{l,u\}} [\text{mm}], \phi\}, D_{mult}$	0, 0, {0.8,1.2,1.0}, 1.0
Packing group	
$S_{RN}, P_m [\text{MPa}], \varepsilon_p, \varepsilon_{lim}, n_{lim}$	10000, 30, 1×10^{-2} , 8×10^{-3} , 2×10^6
C_p, n_c	1, 0.08
Parallel-bonded material group	
Linear group	
$E^* [\text{GPa}], \kappa^*, \mu$	1.5, 1.5, 0.4
Parallel-bond group	
$g_i [\text{mm}], \bar{\lambda}, \bar{E}^* [\text{GPa}], \bar{\kappa}^*, \bar{\beta}$	0, 1.0, 1.5, 1.5, 1.0
$(\bar{\sigma}_c)_{\{m, sd\}} [\text{MPa}], (\bar{c})_{\{m, sd\}} [\text{MPa}], \bar{\phi} [\text{degrees}]$	{10,0}, {100,0}, 0
Linear material group	
$E_n^* [\text{GPa}], \kappa_n^*, \mu_n$	1.5, 1.5, 0.4

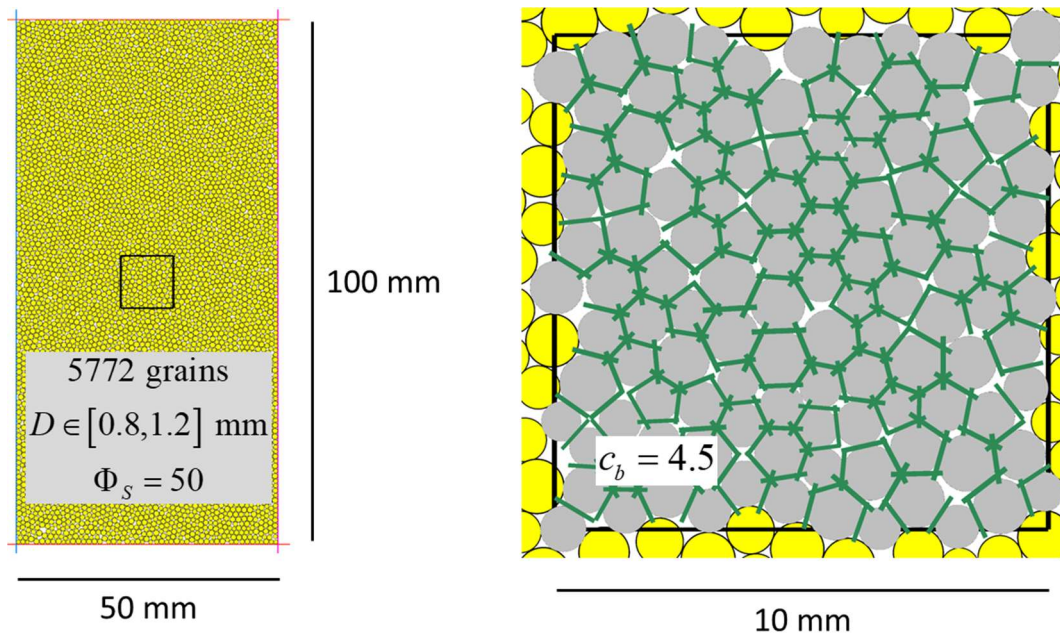


Figure 7 *Base material at the end of material genesis with microstructure consisting of grains and intact parallel bonds (green lines) in the 10 by 10 mm microstructural box.*

The microstructure of the base material is degraded by reducing the width of the cement bridges to 25% of their initial width, removing 50% of the cement at the contacts, and removing 20% of the grains. The microstructures are compared in Figure 8. The bond coordination number decreases with the removal of cement and grains. Direct-tension and UCS tests are performed on the four materials. The stress-strain curves and damage at the end of each test are shown in Figures 9 to 12. Microstructural degradation reduces both the Young's modulus and strengths.

Reducing the width of the cement bridges: (a) reduces the strengths by a factor of 4 for both tests; (b) does not affect the brittle responses (abrupt stress drop after peak load); and (c) does not affect the failure mode in the direct-tension test but changes the failure mode in the UCS test from axial splitting for the base material to shear splitting for the degraded material.

Removing cement at the contacts: (a) reduces the strengths by factors of 28 and 5 for the direct-tension and UCS tests, respectively; (b) does not affect the brittle response in the UCS test but changes the response to brittle followed by some post-peak strength in the direct-tension test; and (c) does not affect the failure mode in the direct-tension test but changes the failure mode in the UCS test from axial splitting for the base material to shear splitting for the degraded material.

Removing some of the grains: (a) reduces the strengths by factors of 5 and 11 for the direct-tension and UCS tests, respectively; (b) does not affect the brittle responses; and (c) does not affect the failure mode in the direct-tension test but changes the failure mode in the UCS test from axial splitting for the base material to crushing near the platens for the degraded material.

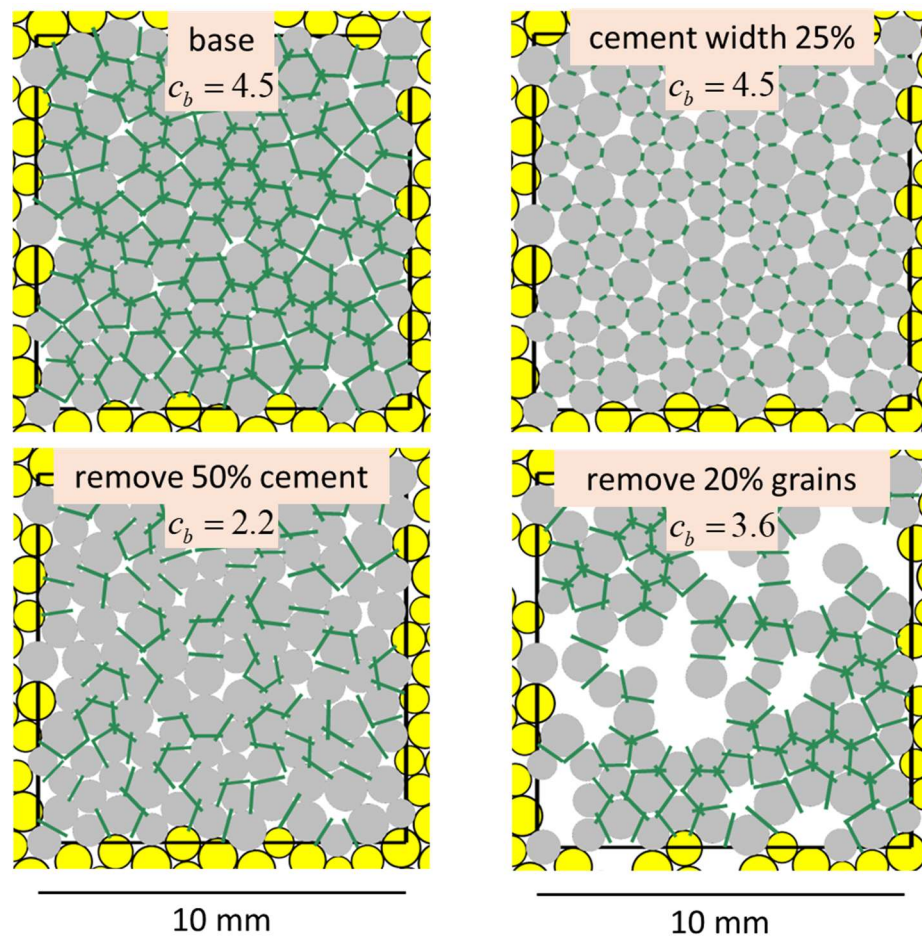


Figure 8 *Microstructures of base and degraded materials.*

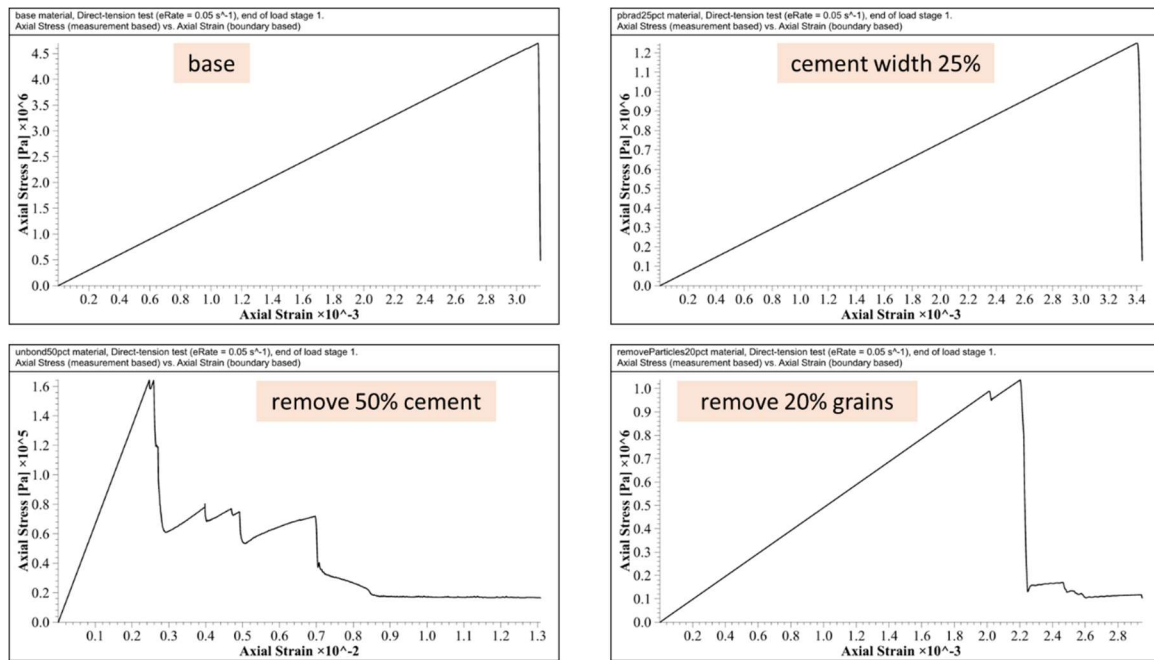


Figure 9 Stress-strain curves for direct-tension tests on base and degraded materials.

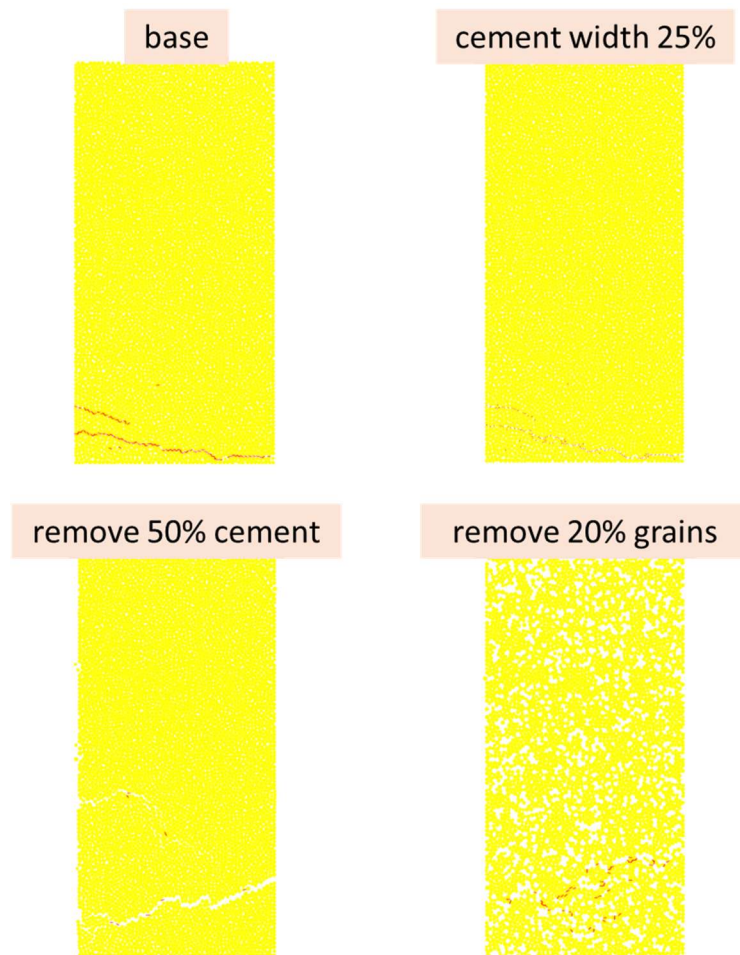


Figure 10 *Damage at end of direct-tension tests on base and degraded materials.*

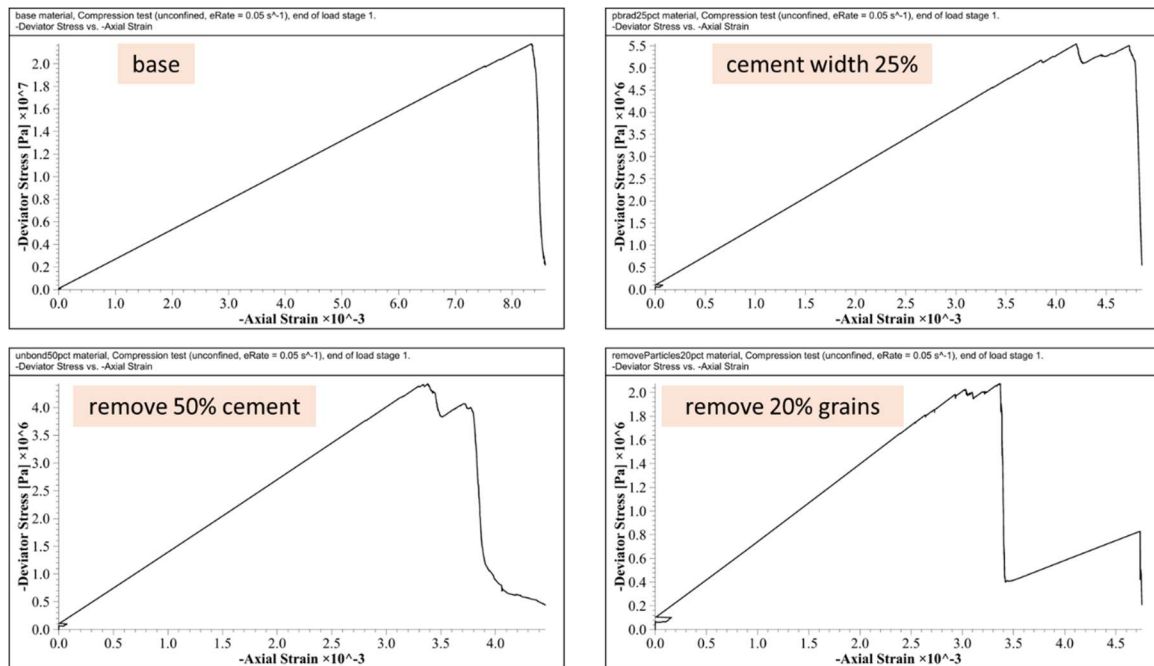


Figure 11 Stress-strain curves for UCS tests on base and degraded materials.

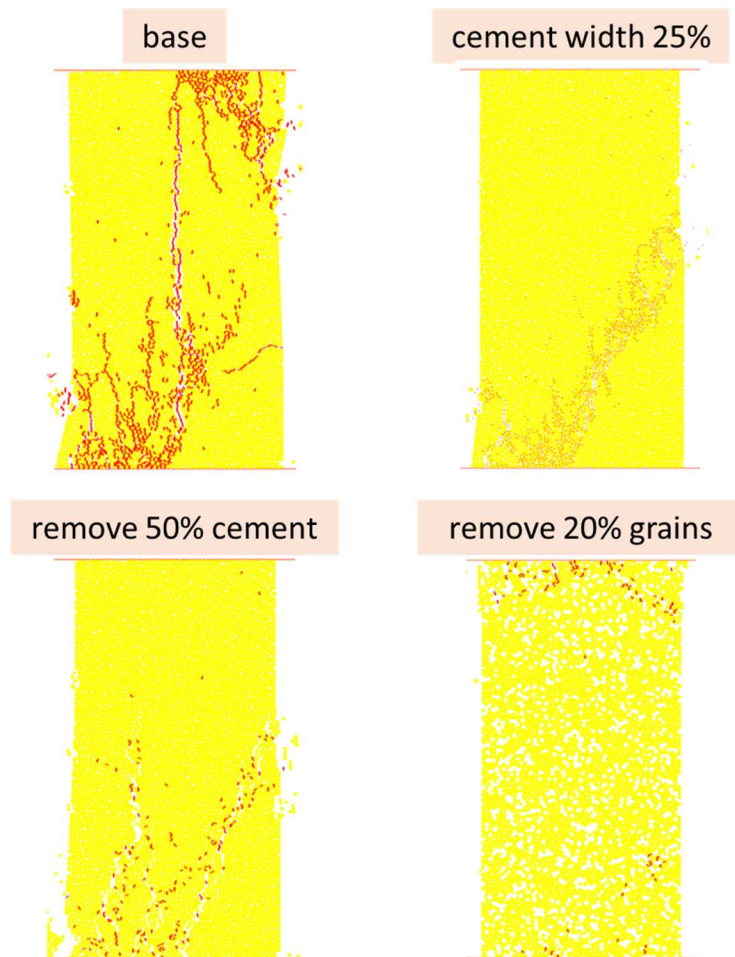


Figure 12 Damage at end of UCS tests on base and degraded materials.

3.0 REFERENCES

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